FLUID FLOW IN A SYSTEM OF PARALLEL CYLINDERS PLACED AT RIGHT ANGLES TO THE FLOW DIRECTION AT LOW REYNOLDS NUMBERS

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Fluid flows in systems of parallel circular cylinders placed at right angles to the flow direction at low Reynolds numbers are of interest for the theory of filtration through fibrous filters. Only theoretical investigations have been reported in the literature. The following expression for the flow function in this system was given in [1]

$$\psi(r,\,\theta)=\frac{U_0\sin\theta}{2}\Big[-\frac{\ln\alpha}{2}+\alpha-\frac{\alpha^2}{4}-0.75\Big]^{-1}\times$$
$$\times\Big\{\Big(1-\frac{\alpha}{2}\Big)\frac{1}{r}-(1-\alpha)\,r+2r\ln r-\frac{\alpha r^3}{2}\Big\}\,, \qquad (1)$$

where r, θ are dimensionless polar coordinates with the origin on the axis of a cylinder, U_0 is the incident flow velocity, α is the fraction of the volume occupied by the cylinders, which is equal to $\pi a^2 n$, and n is the number of cylinders per unit area of the transverse cross section of the system. It was assumed in the derivation of the above formula that the system of cylinders moves with a constant velocity U_0 in a stationary fluid, and therefore the boundary conditions were formulated on the surface of the cylinder. It was also assumed that the flow velocity and the vorticity were zero on the surfaces of cylindrical shells coaxial with the cylinders (with radius b, for which $\pi b^2 n = 1$). This boundary condition ensures that the problem can be solved, but the inaccuracy introduced thereby increases with increasing distance from the surface of the cylinder.

In a similar derivation given in [2] it was assumed that the radial component of the flow velocity and the viscous stress were zero on the shells mentioned above. In this case, the flow function was given by

$$\psi(r,\,\theta)=\frac{U_0\sin\theta}{2}\Big[-\frac{\ln\alpha}{2}+\frac{\alpha^2}{2(1+\alpha^2)}-0.5\Big]^{-1}\times$$
$$\times\Big\{\frac{1}{(1+\alpha^2)\,r}-\frac{(1-\alpha^2)\,r}{1+\alpha^2}+2r\ln r-\frac{\alpha^2 r^3}{1+\alpha^2}\Big\}\,. \qquad (2)$$

The above comments about the accuracy of Eq. (1) are also valid for Eq. (2), and it is difficult to choose between these two formulas on theoretical grounds alone. We note that the streamlines drawn in accordance with these two formulas are practically identical, but the flow velocities are appreciably different because of the difference between the expressions in the brackets. We have carried out a number of experiments to solve this dilemma, and to determine the ranges of validity of the above formulas.

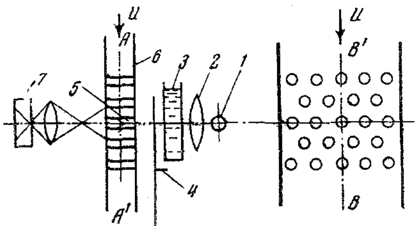

Fig. 1

To visualize the two-dimensional flow it is usual to photograph the trajectories of particles floating on the surface of the flowing fluid. However, meniscus effects distort the trajectories close to the surfaces of the cylinders, and we have therefore determined the trajectories of particles suspended inside the liquid and moving in the median plane of a plane-parallel container.

The apparatus is shown in Fig. 1a, and the disposition of the cylinders is shown in Fig. 1b. Light from the DRSh-250 mercury lamp (1) passes through the set of quartz lenses (2), the heat filter (3), and the shutter (4). It is focused on a conical mirror with an aperture angle of 90°; the mirror, in turn, is located in the central hollow cylinder (5). The light is thus reflected at right angles from this mirror, and illuminates particles suspended in the fluid flowing in the downward direction through the transparent container (6). The particles and the profile of the central cylinder are photographed with a "Start" camera (7) through a KM-8 cathetometer.

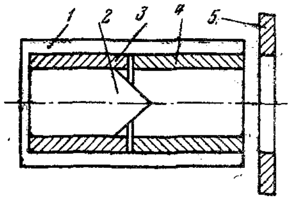

Fig. 2

The design of the central cylinder is illustrated in Fig. 2, where (1) is a transparent cylinder envelope, (2) is the conical mirror, (3) and (4) are blackened tubes, and (5) is a ring.

The width of the illuminated zone in the liquid is determined by the width of the gap between the tubes, which can be adjusted. It is very important to obtain a sharp outline of the cylinder on the photographs. This was achieved as follows. A ring whose inside diameter was equal to the outside diameter of the tubes was placed coaxially in front of the cylinder. A very small amount of light, regulated by the distance between the ring and the tube, then enters the cylindrical shell. The end of the shell was blackened, and a beveled strip 0.2 mm wide was taken off its edges. The image of the cylinder was then in the form of a black circle having a light edge and a sharp external boundary.

The camera was placed at a distance of 0.75-1.5 m from the container. Since the cylinder length was 22 mm, and the diameter 7 and 14 mm, calculations showed that it was possible to photograph all the particles at distances greater than 0.025 and 0.05 mm, respectively, from the surface of the cylinder. The depth of focus was sufficient to produce sharp images of both particles and cylinder.

The experiments were performed in a 95% solution of glycerin with spherical gold-coated 20-μ particles of polymethyl methacrylate. For photographs taken at larger distances from the cylinder, silvered 80-μ particles were used. The particle density was equal to the density of the liquid. The photographs were taken under uninterrupted illumination, the intervals between exposures being in the range 2.7-5.4 sec, and the exposure time 0.2 sec. The flow velocity was 0.06-0.16 cm/sec, which corresponded to Reynolds numbers (referred to the cylinder diameter) of 0.01-0.05. The particle images were circular or elongated spots.

The photographs were analyzed as follows. They were projected onto a dense net of streamlines calculated from Eq. (1) or (2), so that the outline of the cylinder on the photograph and on the drawing coincided, and the flow directions were the same. As can be seen from Fig. 3, the agreement between theory and experiment can be regarded as satisfactory for streamlines passing close to the cylinder (approximately up to r = 2 for α = 0.05, as shown in Fig. 3a, and up to r = 1.5 for α = 0.2, as shown in Fig. 3b).

Knowledge of the incident flow velocity U_0 is necessary for the comparison of the flow velocity calculated from the distance between the particle images at different points of the field with the theoretical results. In the case of a system of cylinders extending laterally to infinity, U_0 is the flow velocity at entry into the system. The flow velocity profile at the entrance to the system was calculated for a rectangular container with cylinders filling its entire cross section

(Fig. 1a). Allowance was made for the fact that the resistance offered by the cylinders was uniformly distributed over the transverse cross section of the container. It was found that the flow velocity was practically constant in the median plane AA'. This conclusion was confirmed by experiment and showed that the narrow container walls (parallel to the cylinders) had no effect on the flow profile in the central part of the container. Hence, it follows that we can assume a parabolic flow profile in the median section BB'.

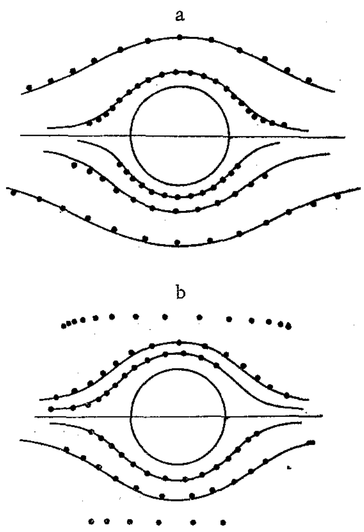

Fig. 3

These considerations have led to the following expression for the flow velocity in the central part of the section AA':

$$U_0 = \frac{1.5\,Q}{S}\left[1 - \frac{\operatorname{th}(h\sqrt{k})}{h\sqrt{k}}\right]^{-1},$$

$$k = \frac{4\alpha}{a^2}\left[-\frac{\ln\alpha}{2} + \alpha - \frac{\alpha^2}{4} - 0.75\right]^{-1},$$

where Q is the flow rate, S is the transverse cross section of the container, and 2h is its width.

From the values of U_0 found in this way we have calculated the flow velocity at different points. Good agreement was found between experimental data and the values calculated from Eq. (1): the constant in the numerator of this expression found experimentally was 0.75 ± 0.02 for both $\alpha = 0.05$ and $\alpha = 0.2$. The formula given by Eq. (2), on the other hand, showed an appreciable discrepancy with experiment.

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